

International Research and Education in Engineering

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ABSTRACT: Liquid fluid drops tend to spherical shapes under equilibrium configurations. When perturbed, these fluid droplets oscillate, representing the competition between the inertia and capillary action of the fluid drop. The qualitative behavior of these drops is similar to the free interfaces of the contact zone of the Planar-flow Melt Spinning device used to continuously cast sheets of metal. In collaboration with scientists from Germany, we have analytically modeled the capillary oscillations of coupled spherical cap droplets. We have also learned about the limitations of current numerical techniques of free interface problems in fluid mechanics. In addition to these academic achievements, we have learned a great deal about the German academic system, culture and formed a productive collaboration with a foreign scientist.

INTRODUCTION

Planar-flow melt spinning (PFMS), also known as planar-flow spin casting (PFSC) is a technique for rapidly freezing metals continuously. The process developed over 20 years ago, can produce either glassy metals or microcrystalline solids depending on the metal being processed and the operating conditions. Research is currently being performed on theoretical and experimental aspects of PFMS, under the current NSF Grant awarded to Dr. Paul Steen at Cornell University. Originally, the IREE project proposal was written with plans for modeling the three-phase contact line made by the solid wheel, molten liquid and atmosphere. However, due to a change in personnel, the project has been modified slightly.

Experimental results show that oscillations of the upstream meniscus lead to surface defects in the cast metal. Understanding the processes that lead to these surface imperfections will allow one to minimize the size and number of such defects, and essentially control the quality of the cast. We consider the oscillations of the meniscus by idealizing the region as coupled spherical caps droplets [1]. The goal of the project is to analytically and numerically study the oscillations of the coupled spherical cap system as a model for the puddle dynamics in the PFMS device. This work is an extension of Theisen et. al. [2], that considers more generalized mode shapes. The idealized system may also be of interest to other applications that display capillary action, such as inkjet printing, crystal growth and liquid lensing.

The broader goals for the traveller, Joshua Bostwick, are to obtain experience on numerical simulations of free interface problems, further generalize previous work on constrained capillary oscillations, develop working relationships with foreign scientists, experience different research environments and learn the German language.

Dr. Peter Ehrhard was selected as a host collaborator, because of his expertise in the field of computational fluid mechanics, interest in free interface problems and long time collaboration with Dr. Paul Steen. He is specifically interested in the numerical aspects of the research problem and would like to use the analytical work as a verification to his numerical computer code. Ehrhard is the chair of the fluid mechanics

department at Dortmund Universitat in Dortmund, Germany, which is located in the German state of North Rhine Westphalia in the manufacturing center of the Ruhr valley.

Bostwick worked in Dortmund, Germany from September 2, 2007 to December 30, 2007.

RESEARCH ACTIVITIES AND ACCOMPLISHMENTS OF THE INTERNATIONAL COOPERATION

The research goal for the trip was to have an analytical and numerical understanding of the capillary oscillations of coupled spherical cap droplets. Developing the analytical model for the system of coupled spherical cap droplets is an extension of previous work on constrained capillary oscillations of a spherical fluid drop. We generalize the previous analysis to include symmetric sub-hemispherical or super-hemispherical base states.

Fluid drops assume spherical shapes under equilibrium conditions. When perturbed, these droplets oscillate reflecting a competition between inertia and surface tension acting on the interface of the droplet. Recently, attention has been paid to spherical fluid drops under a variety of different constraints, because of applications like inkjet printing, crystal growth and liquid lensing. The type of constraint can affect the natural frequency of vibration, as well as the mode shapes in a dramatic way. In the prior mentioned work, we report that pinning a spherical fluid drop on a circle of contact, gives rise to shifting of the eigenfrequencies and changes to the mode shapes. It is also interesting to note that under these constraint conditions; a low frequency mode is activated, which was not present when the constraint is relaxed. This is of practical importance, because the lowest frequency mode is often the first to be excited.

As a natural extension of the previous results, we consider the linear capillary oscillations of symmetric coupled spherical cap droplets pinned on a circle of contact (*see figure 1*). This extension has a practical application to the PFMS device under the current NSF Award. As shown in figure 2, the puddle region, specifically the free interface of the upstream meniscus tends to spherical cap shapes. These capillary oscillations are thought to produce periodic defects in the cast product. Thus an understanding of constrained capillary oscillations of coupled spherical cap droplets will lead to control of such undesirable defects.

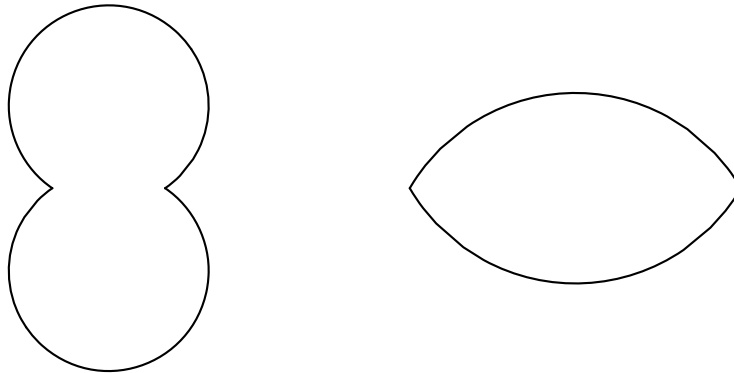


Figure 1. (Superhemispherical and Subhemispherical Base States)

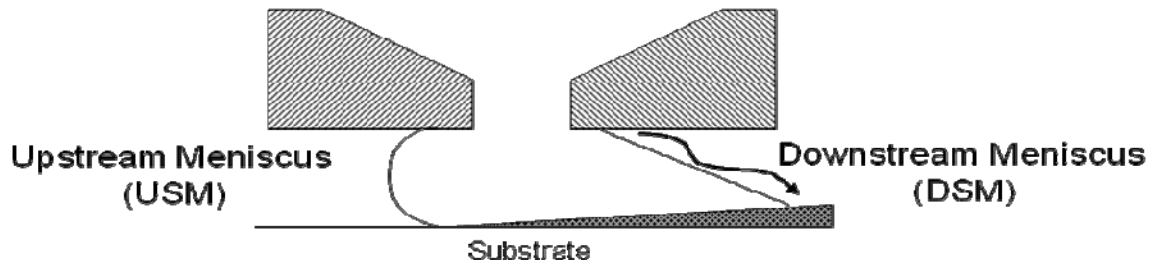


Figure 2 (PFMS Puddle Region)

We use toroidal coordinates as a natural coordinate system to describe both the free interface perturbation, off the spherical cap shape, and the bulk of the spherical cap droplet system. This formulation gives rise to an integro-differential eigenvalue problem on the free surface deformation, subject to constraints on the conservation of volume of the perturbation and the relevant boundary condition on the pinned circle of contact. The problem is reduced to a truncated set of algebraic equations (Galerkin expansion) using a basis of conical functions. From these algebraic equations, a numerical routine is used to compute the eigenfrequencies and eigenmode shapes. Shown below are a few examples of computed eigenmode shapes (*figure 3*).

To simplify the analysis, it is natural to use the symmetry of the problem to consider the odd/even extension of mode shape oscillations for the coupled spherical cap configuration. As a consequence of this assumption, the even extension eigenfrequencies and eigenmodes may be associated with capillary oscillations of a spherical cap droplet resting on a plate. Capillary oscillations under this constraint are also of practical interest.

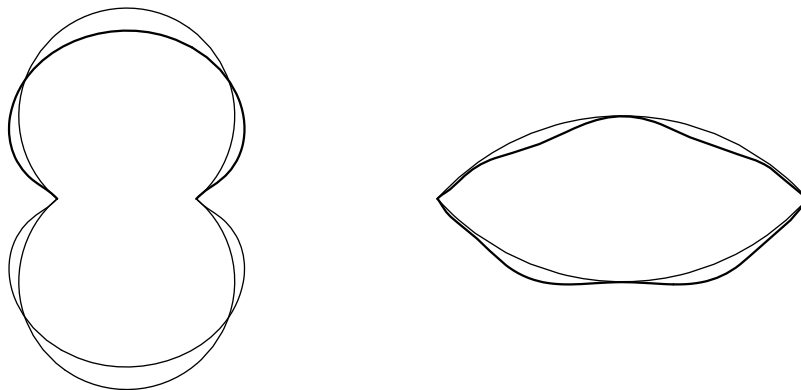


Figure 3(Computed Eigenmode Shapes)

As a second research item, we hoped to use numerics to analyze the oscillations of the constrained spherical fluid drop and coupled spherical cap droplets. The numerical analysis is of interest to us, because we may verify the analytically computed mode shapes and vibrational frequencies. In addition, knowledge of the numerics of free interface flows would be useful if we would start numerically simulating the dynamics of the puddle region in the PFMS device.

This research item is also of interest to Ehrhard, because computer code has been developed by himself and his students to simulate two phase flows with surface tension, or free interface problems. In contrast to our specific objectives, Ehrhard wishes to verify computer code written in Dortmund for two phase flows, or free interface problems in fluid mechanics.

The computer code uses a conservative level set method (Olson [3]) to simulate free interface problems in two phase flows. The interface is represented implicitly by using the 0.5 level set of a function, which can be thought of as a smeared Heaviside function, using 0 and 1 as distinguishing the first and second phase. In contrast to other numerical techniques, which use markers to distinguish the surface, the level set method is able to deal directly with topological changes on the interface.

Prior numerical work on two phase flows from the Dortmund group has included rising bubbles in a chamber, which differ from the constrained oscillations of a sphere in the boundary conditions that need to be applied. The introduction of the constraining circle of contact means, at the pin location, there is a three phase line made by the fluid drop, fluid of immersion and contacting material. Numerical implementation in the presence of this three point contact line proved difficult in many ways, such as: mesh generation, initialization of the level set function for the interface and the boundary conditions for the level set function at the pin location. After discussing these issues, it was decided that the an easier problem to use for the verification of the numerical code was the capillary oscillations of a fluid drop on a plate. As mentioned earlier, the even extension of the coupled spherical cap oscillations gives the Dortmund group an analytical result to verify their numerics.

Ehrhard and his graduate students were able to provide support and direction with respect to the numerical simulations of the constrained fluid drop. The members of the fluid mechanics department in Dortmund served as mentors to Bostwick, because of his limited experience with computational fluid mechanics. The amount of research on the numerics was limited, because a computer for simulations was available for only the last half of the research visit.

Despite the problems associated with the numerical research work, a working knowledge of the conservative level set method using OpenFoam was obtained, which was a goal for the research trip. In addition, the lack of a computer to use for the first half of the trip, allowed more analytical research to be completed. This new analytical work may be useful for the Dortmund group in verifying their numerical code.

BROADER IMPACTS OF THE INTERNATIONAL COOPERATION

The research experience in Germany was very successful. The completed research helped to advance our knowledge of treating overconstrained problems in capillary dynamics, as well as to strengthen the existing working collaboration with Peter Ehrhard.

As a visiting scientist in the fluid mechanics department, Bostwick was able to participate in many departmental projects, in addition to conducting research. These projects include; i) helping to teach a course, ii) reviewing undergraduate capstone projects, iii) participating in interdepartmental competitions iv) departmental parties/excursions and v) taking an introductory course in the German language.

Bostwick co-taught a course titled, “Advanced Transport Phenomenon” to a group of first year graduate students. The fall semester course met twice a week, with the first lecture being taught by Dr. Ehrhard. As a supplement to the formal lecture given by Ehrhard, Bostwick led the second session of the week, which had a problem solving emphasis. This course is a core course to students from Chemical and Biological Engineering. The formal structure of the course differed significantly from the standard American system. The German system encourages students to work more independently by not requiring weekly assignments or giving periodic examinations during the semester. Instead, the students are given an oral examination, upon completion of the semester. The students have the option of postponing the exam, if they feel they are not prepared, and are given multiple chances to pass this exam. This is significantly different than the American system, where students who do not receive a satisfactory grade, must retake the course in its entirety.

Bostwick was able to interact with many students from the course he was teaching, because they were also taking the same German language course. These students came from countries like Venezuela, Pakistan, India and South Korea. These students were able to provide perspective on study/research in their home

countries, as well as another perspective on the German academic system. It was also interesting to hear these foreign students's perception of the American academic system. Many of these students had great respect for American universities and communicated the difficulty for foreign students to study in the United States. From interacting with the students, Bostwick had the impression that students views were mixed with respect to the course/grading and academic structure of the German system.

Periodically throughout the semester, there were events which Bostwick attended to become familiar with the German culture and to interact with fellow graduate students from the university. Within the college of engineering, there was a competition between departments. The competition consisted of monthly events, such as Karaoke night, Texas Hold-Em Tournament and a soccer tournament. Teams from each department competed against one another for points, which are computed at the end of the semester. The department head of the losing department must then host a party for the college of engineering graduate students. These events provided Bostwick an opportunity to meet fellow graduate students, learn about the German culture and practice the German language. Within the fluid mechanics department, there were numerous excursions, such as a bike ride to a neighboring botanical garden and Christmas party at the Dortmund Christmas Market.

The importance of these interactions between Bostwick and the foreign scientists cannot be overstated, with regard to continued research collaborations. The expertise of the fluid mechanics group from Dortmund University is in numerical simulations of free interface/two-phase flows. This is particularly useful with respect to the current NSF award for spin casting at Cornell University, where current work is primarily focused on experimental and analytical analysis.

DISCUSSION AND SUMMARY

Overall, the research experience in Germany was very successful. The completed research helped to advance our knowledge of treating overconstrained problems in capillary dynamics and further generalized previous work with respect to constrained capillary oscillations. These oscillations have direct relevance to defects related to vibration of the upstream meniscus in the PFMS device. Although there was no numerical verification to the analytical results obtained, we learned about the limitations and future direction of research with regard to numerical analysis of two-phase flows, specifically for problems with a three phase contact line.

In addition to the academic achievements, the trip was beneficial in gaining perspective on the German culture, developing working relationships with German scientists and observing a foreign research/working environment.

ACKNOWLEDGEMENTS

The authors would like to acknowledge support from the National Science Foundation grant, NSF-0423791.

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BRIEF BIOGRAPHIES OF RESEARCHERS

Joshua Bostwick received a B.S.E. degree in Civil Engineering & Mechanics with a minor in Mathematics and a B.S. in Physics from the University of Wisconsin-Milwaukee in 2005. Currently, he is working on his PhD in Theoretical and Applied Mechanics from Cornell University under the direction of Dr. Paul Steen. His research interests include capillary oscillations, free interface dynamics, stability theory, symmetry methods applied to dynamical systems and engineering education.

Dr. Paul H. Steen has been at Cornell since 1982. He is a Professor in the School of Chemical and Biomolecular Engineering, with field affiliations in Applied Mathematics and Theoretical and Applied Mechanics. His research is in the area of dynamics and stability of fluid systems with interfaces. Current focus is on shape-changes of gas/liquid and liquid/liquid interfaces and stability issues arising in the continuous casting of thin sheets of metal. He is a fellow of the American Physical Society (1996) and has been active in APS/Division of Fluid Dynamics affairs as chair of the Fluid Dynamics Prize Committee, and as member of the Executive, Program, Publications and Frenkiel Award Committees. He has co-edited "A Gallery of Fluid Motion", a DFD-APS project published by Cambridge University Press. He is an Associate Editor of the Journal of Fluid Mechanics. He has more than 60 journal publications and has edited several books. Prior to coming to Cornell, Steen received his PhD from The Johns Hopkins University in 1981 and held post-doctoral position in Chemical Engineering at Stanford University, after having completed undergraduate degrees in Engineering and English Literature at Brown University. At Cornell, he has served as Director of Graduate Studies for Chemical Engineering. He has received an Alexander von Humboldt Fellowship and has been a Senior Guest Scientist at the Forschungszentrum Karlsruhe, Germany.